16MSPCH302 — NUCLEAR AND PARTICLE PHYSICS

INTRODUCTION TO NUCLEAR REACTIONS

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1 A DESCRIPTION OF NUCLEAR REACTIONS

NUCLEAR REACTIONS GENERALLY involve two reactants which are simply two particles bombarding each other. Although this can just as well be two moving particles coming in contact nuclear reactions are often simplified to one stationary particle and one moving particle bombarding the stationary 'target'. Think back to the first experiment with which we started our discussions of nuclei: Rutherford's experiment bombarding a gold film with He^4 particles is a nuclear reaction.

It is worth noting that the electron scattering experiment is a reaction of type $a + X \rightarrow X + a$ since the incoming protons remain protons after collision and the gold target too stays put. Reactions of the type $X(a, a)X$ are called nuclear scattering.

Let us talk about generic nuclear reactions where a particle a bombards the target X resulting in some particles b and Y. That is to say the nuclear reaction gives rise to new particles by changing the target and the incoming particle or creates new particles through mass-energy equivalence. Keep in mind that the lowercase a and b refer to particles that are traveling and the uppercase X and Y refer to particles that are – at least relatively – stationary. In a way this represents scattering.

We will assume, unless otherwise stated, that both a and X are positive. Undoubtedly a could also be an electron, but we will assume for uniformity that it is positively charged. The target nucleus X is unquestionably positively charged. The formal representation of a reaction like $a + X \rightarrow b + Y$ is simply $X(a, b)Y$ and we will start using this shorthand wherever convenient from this point onwards.

Finally, despite the similarities in their structure, with there being reactants and products, nuclear reactions must not be confused with chemical reactions which (besides largely being in the domain of chemists and not us physicists) sees a change in the electronic arrangement rather than in the arrangement of nucleons.

The reaction $X(a, \gamma)Y$ is a commonly encountered one called radiative capture; likewise the type $X(\gamma, n)Y$, $X(\gamma, p)Y$ or $X(\gamma, \alpha)Y$ is also a common type of nuclear reaction known as photodisintegration. This involves the absorption of energy and is a common occurrence in massive supernovae; it dampens the energy available to a star causing to its collapse into either a neutron star or a black hole. The basic idea behind photodisintegration is reiterated in §3.

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2 ENERGETICS

2.1 *Q value*

Every reaction involves the use or release of some energy. Some reactions require energy to occur (**endothermic**) while others release energy (**exothermic**) when they occur. This is an idea that agrees with chemical reactions in general.

The energy of a nuclear reaction stems from mass-energy equivalence $E = mc^2$. Suppose we represent the rest mass energy of an element with a prime, the reaction $X(a, b)Y$ sees the following:

$$m'_a c^2 + \frac{m_a v_a^2}{2} + m'_X c^2 + \frac{m_X v_X^2}{2} = m'_b c^2 + \frac{m_b v_b^2}{2} + m'_Y c^2 + \frac{m_Y v_Y^2}{2}$$

with the left-hand side representing the reactants and the right-hand side representing the products. Subsequently,

$$(m'_a + m'_X - m'_b - m'_Y)c^2 = T_b + T_Y - T_a - T_X$$
$$(\Delta m)c^2 = Q = T_f - T_i \tag{1}$$

Endothermic reactions are also known as 'endoergic' reactions; exothermic reactions are likewise also known as 'exoergic' reactions.

where Q is called the **Q value** of the equation $X(a, b)Y$ and tells us excess or deficit of kinetic energy (represented here by T with the subscript f for final and i for initial). This gives a mathematical description for exothermic and endothermic reactions: in the former energy is released making $T_f > T_i$ and, therefore, $Q > 0$; in the latter the reactants take in more energy than the reaction finally gives out making $T_f < T_i$ and hence $Q < 0$. The energies fuelling a nuclear reaction goes towards overcoming the binding energy of the participating nuclei.

2.2 *Threshold energy*

We can make our solution a little more accurate. As discussed earlier the problem comes down to relative motion where one of the reactants is considered stationary (the target) and the other is said to bombard it along some line aX, naturally with some linear momentum. Say the two products b and Y are displaced such that b makes an angle θ with the horizontal aX and Y makes an angle φ with the same aX. Say the products b and Y have momenta p_b and p_Y respectively. Their resultant momenta in the aX direction, by simple vector resolution, will then be $p_b \cos\theta$ and $p_Y \cos\varphi$. And since the net momentum of the system must be conserved we end up with

$$p_b \cos\theta + p_Y \cos\varphi = p_a \tag{2}$$

The corresponding equation along the vertical, perpendicular to aX, is simply

$$p_b \sin\theta + p_Y \sin\varphi = 0 \tag{3}$$

which is equal to zero since there is no fully vertical displacement among the products. Observe that the parameters T_μ and p_μ of a particle μ are equivalent i.e. they behave alike so knowing one will tell us quite a lot about the general behaviour of the other. Here our focus is clearly on p_μ but, previously, we had looked at T_μ as well.

In most nuclear reactions the product Y (or b if you like) is only theoretical. We can, without betraying empirical observations, get rid of all terms representing Y in eq. (1), (2) and (3). This leaves us with the following equations:

$$\mathrm{Q} + \mathrm{T}_a = \mathrm{T}_b \qquad\qquad p_a = p_b$$

The fact that $\mathrm{T}_b - \mathrm{T}_a - \mathrm{Q} = 0$ and $p_a - p_b = 0$ means we can safely multiply both of these by arbitrary quantities before equating them.

$$\begin{aligned}
m_{\mathrm{Y}}(\mathrm{T}_b - \mathrm{T}_a - \mathrm{Q}) &= \frac{p_a^2 - p_b^2}{2} \\
&= \frac{-p_a^2 - p_b^2}{2} + p_a^2 \\
&= \frac{-p_a^2 - p_b^2}{2} + p_a p_a \\
&= \frac{-p_a^2 - p_b^2}{2} + p_a p_b \cos\theta \\
\frac{m_{\mathrm{Y}}}{2}(\mathrm{T}_b - \mathrm{T}_a - \mathrm{Q}) &= \frac{-p_a^2}{4} - \frac{p_b^2}{4} + \frac{p_a p_b \cos\theta}{2} \\
\frac{m_{\mathrm{Y}}\mathrm{T}_b - m_{\mathrm{Y}}\mathrm{T}_a - m_{\mathrm{Y}}\mathrm{Q}}{2} &= \frac{-m_a\mathrm{T}_a}{2} - \frac{-m_b\mathrm{T}_b}{2} + \sqrt{m_a m_b \mathrm{T}_a \mathrm{T}_b}\cos\theta \\
0 &= \left[\frac{m_{\mathrm{Y}} + m_b}{2}\right]\mathrm{T}_b - \cos\theta\sqrt{m_a m_b \mathrm{T}_a \mathrm{T}_b} - \left[\frac{m_{\mathrm{Y}}\mathrm{Q} + m_{\mathrm{Y}}\mathrm{T}_a - m_a\mathrm{T}_a}{2}\right]
\end{aligned}$$

This is a quadratic equation in T_b which we can solve using the quadratic formula. We do not have to solve it completely since the nature of T_b becomes evident from simple substitution of a, b and c into the equation from the format $ax^2 + bx + c = 0$, i.e.

We have used $m\mathrm{T} = p^2/2$ several times in this derivation.

$$\sqrt{\mathrm{T}_b} = \frac{\sqrt{m_a m_b \mathrm{T}_a}\cos\theta \pm \sqrt{m_a m_b \mathrm{T}_a \cos^2\theta + (m_{\mathrm{Y}} + m_b)[m_{\mathrm{Y}}\mathrm{Q} + (m_{\mathrm{Y}} - m_a)\mathrm{T}_a]}}{m_{\mathrm{Y}} + m_b} \tag{4}$$

What happens to this equation for a straightforward reaction without deviation from the horizontal of approach? Clearly $\theta = \varphi = 0$ since there is no wastage of energy in imparting non-linear motion. Therefore $\cos\theta = 1$ and the value of T_a becomes

$$\mathrm{T}_a = -\mathrm{Q}\left(\frac{m_{\mathrm{Y}} + m_b}{m_{\mathrm{Y}} + m_b - m_a}\right) = \mathrm{T}_{\mathrm{th}} \tag{5}$$

known as the **threshold energy** of an $\mathrm{X}(a, b)\mathrm{Y}$ reaction. It is only meaningful for endothermic reactions since they need a minimum energy to drive them. Beyond this $\mathrm{T}_a \to \mathrm{T}_a'$ making the value of energy $\mathrm{T}_a' = -\mathrm{Q}m_{\mathrm{Y}}/(m_{\mathrm{Y}} - m_a)$ or the entire range $\Delta\mathrm{T} = \mathrm{T}_a' - \mathrm{T}_{\mathrm{th}}$ which can lead to this becoming a double-valued function.

For more on threshold energy see Krane pp. 384–388.

(Krane, K.S. 'Introductory Nuclear Physics'. John Wiley & Sons, 1988.)

3 PHOTODISINTEGRATION

Reactions of the type (γ, i) where i may be a neutron, a proton or an alpha particle is called **photodisintegration** and involves nuclei absorbing a high-energy γ-ray, getting excited, and almost instantly decaying by giving off a particle i as above. In heavy nuclei the energy given off can be greater than the incoming γ-ray making the process exoergic, else – especially for lighter nuclei – photodisintegration is endoergic.